

# Mathematics

## Choice based credit system (CBCS)

### Course structure

Sl. No.	Paper & Title	Credit	No of Hrs/week Theory/ Prctical	Duration of exam in Hrs Theory/ Prctical	IA Marks Theory/ Prctical	Marks at the Exams	Total Marks
<b>I Semester</b>							
1.1	Foundations of Analysis	4	4	3 Hrs	20	80	100
1.2	Algebra – I	4	4	3 Hrs	20	80	100
1.3	Ordinary Differential Equations	4	4	3 Hrs	20	80	100
1.4	Real Analysis	4	4	3 Hrs	20	80	100
1.5	Functions of Several Variables	4	4	3 Hrs	20	80	100
1.6	Topology	4	4	3 Hrs	20	80	100
<b>II Semester</b>							
2.1	Complex Analysis	4	4	3 Hrs	20	80	100
2.2	Linear Algebra	4	4	3 Hrs	20	80	100
2.3	Algebra – II	4	4	3 Hrs	20	80	100
2.4	Partial Differential Equations	4	4	3 Hrs	20	80	100
2.5	Classical Mechanics	4	4	3 Hrs	20	80	100
2.6	Open Elective Course – I	4	4	3 Hrs	20	80	100

III Semester							
3.1	Measure Theory and Integration	4	4	3 Hrs	20	80	100
3.2	Discrete Mathematical Structures	4	4	3 Hrs	20	80	100
3.3	Differentiable Geometry	4	4	3 Hrs	20	80	100
3.4	Numerical Analysis	4	4	3 Hrs	20	80	100
3.5	Algebraic Topology	4	4	3 Hrs	20	80	100
3.6	Open Elective Course – II a. Statistics & Quantitative Techniques b. Optimization Techniques	4	4	3 Hrs	20	80	100
IV Semester							
4.1	Functional Analysis	4	4	3 Hrs	20	80	100
4.2	Probability Theory	4	4	3 Hrs	20	80	100
4.3	Differential Manifolds	4	4	3 Hrs	20	80	100
4.4	Optional / Specialization I. Fluid Mechanics II. Number Theory and Cryptology III. Commutative Algebra IV. Mathematical Physics V. Galois Theory VI. Computational Complexity Theory	4	4	3 Hrs	20	80	100
4.5	Optional / Specialization I. Mathematical Finance II. Operations Research III. Graph Theory IV. Fourier Analysis V. Banach Algebra VI. Mathematical Modeling	4	4	3 Hrs	20	80	100
4.6	Project	4	The candidate shall submit a dissertation carrying 80 marks and appear for viva-voce carrying 20 marks				100
	Total	96					2400

**M.Sc. MATHEMATICS UNDER (CBCS)**  
**I SEMESTER**  
**1.1 FOUNDATIONS OF ANALYSIS**

**Objective:**

This Course will give the students, fundamentals of abstract Mathematical ideas required of them and necessary logical foundations.

**Unit 1:**

Peano axioms, Natural numbers, Properties of natural numbers as a well ordered set, Finite sets and their properties, Infinite sets, countable and uncountable sets Examples.

**Unit 2:**

Cardinal numbers and its arithmetic, Schroeder-Bernstein theorem, Cantor's theorem and continuum hypothesis

**Unit 3:**

Zorn's lemma, Axiom of choice and well ordering principle and their equivalence

**Unit 4:**

The completeness Property of  $\mathbb{R}$ : The Least Upper Bound Property (LUB Property) and the Greatest Lower Bound Property (GLB Property). Archimedean Property.

**Unit 5:**

The existence of  $\sqrt{2}$ , Density of Rational Numbers, Nested Interval Property, Weierstrass Theorem, Heine-Borel Theorem

**REFERENCES:**

1. P.R.Halmos, Naive Set Theory, UTM, Springer International.
2. Y.F.Lin and S.Y.T.Lin, Set Theory. An Intuitive Approach, Houghton Mifflin Company
3. I.H.Cohen and Ehrlich, Structure of Real Number Systems, D.Van Nostrand Company
4. Claude.W Burrill, Foundations of Real Numbers, Tata McGraw Hill.



## 1.2 ALGEBRA – 1

### Objective:

The course shall provide algebraic abstractions to the students to understand structures of Mathematical systems in general.

### Unit 1:

Division algorithm, HCF, LCM, Euclid's Algorithm, Fundamental theorem of Arithmetic, Congruence, Chinese remainder theorem, Euler phi function, Group, Subgroup, Normal subgroup and Quotient group

### Unit2:

Group homomorphism, Isomorphism theorems and the correspondence theorem, Center and Commutator subgroup of a group, cyclic group, Lagrange theorem.

### Unit3:

Euler's and Fermat's theorems as consequences of Lagrange's theorem, Symmetric group  $S_n$ . Structure theorem for symmetric groups, Action of a group on a set, Examples, orbit and stabilizer of an element.

### Unit 4:

Class equation for a finite group, Cauchy theorem for finite groups, Sylow theorems, Applications, Wilson's theorem.

### Unit 5:

Subnormal series for a group, Solvable group, Solvability of  $S_n$ . Composition series for a group. Jordan-Holder theorem

### REFERENCES

1. J.B.Fraleigh, Abstract Algebra, Narosa Publications
2. Joseph A. Gallian, Contemporary Abstract Algebra, Narosa Publications
3. N.S.Gopalakrishnan, University Algebra,
4. I.N.Herstein, Topics in Algebra, Wiley
5. Mukopadhyaya and M.K.Sen, Ghosh Shamik, Topics in Abstract Algebra, University Press
6. I.B.S.Passi and I.S.Luther, Algebra Vol-I, Narosa Publications

## 1.3 ORDINARY DIFFERENTIAL EQUATIONS

### Objective:

Differential equations are regarded as the most effective models in understanding physical phenomena. This course will provide at introductory level the essential foundations required.

### Unit 1:

Linear-differential equation of  $n^{\text{th}}$  order differential equation, fundamental sets of solution, Wronskian – Abel's Identity, theorem on linear dependence of solutions, Adjoint, self-adjoint linear operator, Green's formula.

### Unit 2:

Adjoint equations, the  $n^{\text{th}}$  order non-homogenous linear equations. Variation of parameters-zeros of solutions, comparison and separation theorem, Fundamental existence and uniqueness theorem, dependence of solution on initial conditions, existence and uniqueness for higher order system of differential equations.

### Unit 3:

Eigen value problems, Sturm-Liouville's problem, Orthogonality of Eigen functions, Eigen functions, expansion in a series of orthogonal functions, Green's function method, Eigen value problems Sturm-Liouville problems Orthogonality of eigen functions - Eigen function expansion in a series of orthonormal functions- Green's function method.

### Unit 4:

Power series solution of linear differential equations- ordinary and singular points of differential equations, Classification into regular and irregular singular points, series solution for Bessel's and Legendre differential Equations.

### Unit 5:

Series solution about an ordinary point and a regular singular point – Frobenius method-Hermite, Lagrange, Chebyshev and Gauss Hypergeometric equations and their general solutions. Generating function, Recurrence relations, Rodrigue's formula-Orthogonality properties. Behavior of solution at irregular singular points and the point at infinity.

### REFERENCES

1. E. Coddington, Introduction to Ordinary Differential Equations.
2. G. F. Simmons, Introduction to Differential Equations, Tata McGraw.
3. Boyce and DiPrima, Elementary Differential Equations and Boundary Value Problems, J.Wiley.
4. M.S.P. Eastham, Theory of ordinary differential equations, Van Nostrand, London, 1970.
5. S.L. Ross, Differential equations (3rd edition), John Wiley & Sons, NewYork, 1984.



## 1.4 REAL ANALYSIS

### Objective:

Analysis comes closer to reality. This course introduces the notion of metric space on real line in order to define and discuss topics like continuity differentiability and other aspects of one variable functions. Real-function theory and Integration is a mainstay of the course.

### Unit 1:

Metric spaces, Basic definition, Compactness, connectedness, sequences, subsequences and Cauchy sequences in a metric space  $\mathbb{R}$  as a complete metric space Limit, continuity and connectedness, Kinds of discontinuities Algebraic completeness of the complex field.

### Unit 2:

Differentiation Mean value theorems, the continuity of derivatives, Derivatives of higher orders. Taylor's theorem, Analytic functions. Functions of class  $C$  (which is not analytic).

### Unit 3:

Riemann- Stieltjes integral, its linearity, the integral as a limit of sum, change of ions of bounded variables. Mean Value Theorems.

### Unit 4:

Functions of bounded variation, the fundamental theorem of calculus

### Unit 5:

Absolute and conditional convergence of series, Riemann's derangement theorem, Sequences and series of functions, Uniform convergence, Uniform convergence and continuity, Uniform convergence and differentiation. The Stone-Weirstrass theorem

### REFERENCES:

1. Apostol T.M- Introduction to Mathematical Analysis,
2. W.Rudin, Introduction to Mathematical Analysis, Wiley.
3. Terence Tao, Analysis- I and Analysis- II, TRIM series, HBA.
4. Richard,Goldberg, Real Analysis, Oxford and IBH.
5. S.R.Ghorpade and B.V.Limaye, A Course in Calculus and Real Analysis, UTM, Springer

## 1.6 TOPOLOGY

### Objectives:

This course introduces students with the structure of an abstract metric space and its generalization. Geometrical objects can be viewed with this knowledge so that in tandem, general geometrical spaces can be viewed.

### Unit 1:

Separation Axioms : regular and  $T_3$  spaces, normal and  $T_4$  spaces, Urisohn's Lemma, Tietze's, Extension Theorem, completely regular and Tychonoff spaces, completely normal and  $T_5$  spaces.

### Unit 2:

Countability Axioms: First and Second Axioms of countability. Lindel of spaces, seperable spaces, countably compact spaces, Limit point compact spaces

### Unit 3:

Convergence in Topology: Sequences and Sub sequences, convergence in topology. Sequential compactness, one point compactification, Stone – Cech compactification

### Unit 4:

Metric Spaces and Metrizability: Seperation and countability axioms in metric spaces, convergence in metric spaces, complete metric spaces.

### Unit 5:

Product spaces: Arbitrary product spaces, product invariance of certain separation and countability axioms. Tychnoff's Theorem, product invariance of connectedness.

### REFERENCES:

1. J.R.Munkers : Topology –A first course, PHI(2000)
2. M.A.Armstrong, Basic Topology
3. James Dugundji :Topology, PHI(2000)
4. J.L.Kelley : General Topology, Van Nostrand (1995).

## 1.6 Functions of Several Variables

**Objective:**

Having understood analysis on the real line, this course shall generalize the calculus ideas to the functions of several variables. The same notions are revisited under multivariable setting. Classical theorems like inverse function theorem, Implicit function theorem, Stokes and Green's theorem are studied.

**Unit 1:**

Functions of several variables, Directional derivative, Notion of differentiability, Total derivative

**Unit 2:**

Jacobian, Chain rule and Mean-value theorems, Interchange of the order of differentiation.

**Unit 3:**

Higher Derivatives, Taylor's theorem, Inverse Mapping theorem, Implicit function theorem, Extremum problems with constraints,

**Unit 4:**

Lagrange multiplier method, Curl, Gradient, Divergence, Laplacian

**Unit 5:**

Cylindrical and spherical coordinates, line integrals, surface integrals, Theorem of Green, Gauss and Stokes.

**REFERENCES**

1. Apostol T.M- Mathematical Analysis(Ch.6,7,10 and 11)
2. Apostol T.M,Calculus-2-Part 2(Non-Linear Analysis)
3. Vector Analysis (Schaum Series)



## 2.1 Complex Analysis

### Objectives:

This course introduces the analysis to be done on the complex plane. Algebraically  $\mathbb{C}$  being closed, this kind of analysis is very rich in reflecting geometry and topology of the plane for developing similar ideas of real analysis.

### Unit-1:

Complex plane its algebra and topology, Holomorphic maps, Analyticity.

### Unit-2:

Review of Complex integration, behavior of an analytic function in the neighbourhood of a singularity, Argument Principle, Rouché's Theorem.

### Unit-3:

Maximum modulus Principle, Hadamard three circle theorem and their consequences

### Unit-4:

Mittag Leffler's Theorem, Schwartz Lemma, Conformal mapping, Linear transformations.

### Unit-5:

Normal families, Montel's theorem and Riemann Mapping theorem.

### REFERENCES:

1. L.Ahlfors, Complex Analysis, McGraw Hill.
2. J.B.Conway, Functions of One complex variable, Springer.
3. Greene,Robert.F,S.Krantz, Functions of One Complex variable, Universities Press.

## 2.2 Linear Algebra

### Objectives:

This course introduces vector spaces and maps between them. Vector spaces are very important algebraic structures especially for clarifying multivariable notions of geometrical nature. Applications of operators are plenty both in pure as well as applied mathematics.

### Unit1:

Definition and examples of vector spaces, subspaces, Sum and direct sum of subspaces. Linear span, Linear dependence, independence and their basic properties. Basis, Finite dimensional vector spaces. Existence theorem for bases, Invariance of number of elements of a basis set. Dimension, Existence of complementary subspace of a finite dimensional vector space, Dimension of sums of subspaces. Quotient space and its dimension.

### Unit 2:

Linear transformations and their representation as matrices. The algebra of Linear Transformations. The rank nullity theorem. Change of basis. Dual space, Bidual space and natural isomorphism, Adjoint of linear transformation.

### Unit 3:

Eigen values and eigenvectors of a linear transformation, Diagonalization. Annihilator of a subspace. Bilinear, Quadratic and Hermitian forms.

### Unit 4:

Solutions of homogeneous systems of linear equations. Canonical forms, Similarity of linear transformations. Invariant subspaces, Reduction to triangular forms.

### Unit 5:

Nilpotent transformations, Index of nilpotency. Invariants of a linear transformation, Primary decomposition theorem. Jordan blocks and Jordan forms. Inner product spaces;

### REFERENCES:

1. Hoffman and Kunze, Linear Algebra
2. N.Herstein, Topics in Algebra, Wiley Eastern Ltd, New York (1975)
3. S.Lang, Introduction to Linear Algebra 2nd Edition Springer-Verlag (1986)
4. Greub, Werner, Linear Algebra, Universities Press.



## 2.3 Algebra-2

### Objective:

This course is an extension of algebra-I where the students will be exposed to structures like Rings, fields, Integral domains etc. Also they would appreciate ideas like field extensions and number fields.

### Unit 1:

Rings, subrings, ideals, factor ring (all definitions and examples. Homomorphism of Rings, Isomorphism theorems. Correspondence theorem. Integral domain, field and embedding of an integral domain in a field. Prime ideal, maximal ideal of a ring. Polynomial ring  $R(X)$  over a Ring in an indeterminate  $X$ .

### Unit 2:

Principal Ideal Domain (PID). Euclidean domain. The ring of Gaussian integers as an Euclidean domain. Fermat's theorem. Unique factorization domain. Primitive polynomial. Gauss lemma.

### Unit 3:

$F(X)$  is a unique factorization domain for a field. Eisenstein's criterion of irreducibility for polynomials over a unique factorization domain.

### Unit 4:

Field, subfield, prime fields-definition and examples Characteristic of a field Characteristic of a finite field. Field extensions, Algebraic extension. Transitivity theorem. Simple Extensions

### Unit 5:

Roots of Polynomials. Splitting field of a polynomial. Existence and uniqueness theorems. Existence of a field with prime power elements.

### REFERENCES:

1. N.S.Gopalakrishna University Algebra, New Age International Publishers
2. Joseph A. Gallian, Contemporary Abstract Algebra, Narosa Publications
3. I.N.Herstein, Topics in Algebra 2nd Edition, John -wiley and sons, New York
4. Surjit Singh and Quazi Zameeruddin, Modern Algebra, Vikas Publishers (1990)
5. S.K.Jain, P.B.Bhattacharya and S.R.Nagpaul, Basic Abstract Algebra, Cambridge University Press.
6. Mukhopadhyaya and Sen, Modern Algebra, University Press



## 2.4 Partial Differential Equations

### Objectives:

Partial Differential Equations come up while dealing with the analysis of functions involving more than one independent variable. A nonlinear phenomenon is generally modeled as a PDE. The student will be introduced to this subject mainly to understand how a complex phenomenon evolves.

### Unit 1:

First order Partial Differential Equations, the classification of solutions-Pfaffian differential equations-quasi linear equations, Lagrange's method-compatible systems, Charpit's method, Jacobi's method, integral surfaces passing through a given curve.

### Unit 2:

Method of Characteristics for quasi-linear and non-linear equations, Monge cone, characteristic strip.

### Unit 3:

Origin of second order partial differential equations, their classification, wave equation-D'Alembert's solution, vibrations of a string of finite length, existence and uniqueness of solution-Riemann's Method.

### Unit 4:

Laplace equation boundary value problems, Maximum and minimum principles, Uniqueness and continuity theorems, Dirichlet problem for a circle, Dirichlet problem for a circular annulus, Neumann problem for a circle, Theory of Green's function for Laplace equation.

### Unit 5:

Heat equation, Heat conduction problem for an infinite rod, Heat conduction in a finite rod existence and uniqueness of the solution Classification in higher dimensions, Kelvin's inversion theorem, Equipotential surfaces

### REFERENCES

1. I.J. Sneddon, Partial Differential equations, McGraw Hill.
2. F. John, Partial Differential Equations, Springer.
3. P. Prasad, R. Ravindran, Introduction to Partial Differential Equations, New Age International
4. T. Amarnath, An Elementary Course on Partial differential Equations, Narosa Publishers.

## 2.5 Classical Mechanics

### Unit 1:

Coordinate transformations, Cartesian tensors, Basic Properties, Transpose, Symmetric and Skew tensors, Isotropic tensors, Deviatoric Tensors, Gradient, Divergence and Curl in Tensor Calculus, Integral Theorems.

### Unit 2:

Continuum Hypothesis, Configuration of a continuum, Mass and density, Description of motion, Material and spatial coordinates, Translation, Rotation, Deformation of a surface element, Deformation of a volume element, Isochoric deformation, Stretch and Rotation, Decomposition of a deformation, Deformation gradient, Strain tensors, Infinitesimal strain, Compatibility relations, Principal strains.

### Unit 3:

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Material and Local time derivatives Strain, rate tensor, Transport formulas, Stream lines, Path lines, Vorticity and Circulation, Stress components and Stress tensors, Normal and shear stresses, Principal stresses.

### Unit 4:

Fundamental basic physical laws, Law of conservation of mass, Principles of linear and angular momentum, Equations of linear elasticity, Generalized Hooke's law in different forms, Physical meanings of elastic moduli, Navier's equation.

### Unit 5:

Equations of fluid mechanics, Viscous and non-viscous fluids, Stress tensor for a non-viscous fluid, Euler's equations of motion, Equation of motion of an elastic fluid, Bernoulli's equations, Stress tensor for a viscous fluid, Navier-Stokes equation.

### REFERENCE BOOKS

1. D.S. Chandrasekharaiah and L. Debnath: Continuum Mechanics, Academic Press, 1994.
2. A.J.M. Spencer: Continuum Mechanics, Longman, 1980.
3. Goldstein, Classical Mechanics, Addison – Wesley, 3<sup>rd</sup> Edition, 2001.
4. P. Chadwick : Continuum Mechanics, Allen and Unwin, 1976.
5. Y.C. Fung, A First course in Continuum Mechanics, Prentice Hall (2<sup>nd</sup> edition), 1977
6. A.S. Ramsey, Dynamics part II, the English Language Book Society and Cambridge University Press,(1972)
7. F. Gantmacher, Lectures in Analytical Mechanics, MIR Publisher, Mascow, 1975.
8. Narayan Chandra Rana and Sharad Chandra Joag, Classical Mechanics, Tata McGraw Hill, 1991.
9. F. Chorlton, Text Book of Dynamics, (ELBS Edition), G. Van Nostrand and co.(1969).



## 2.6 OEC

### SET THEORY (A LANGUAGE OF MATHEMATICS)

#### Objective:

This course is offered to the students who have diverse background. Enough care is taken to see that the students of this course would familiarize themselves with the language of Mathematics through set theory.

#### Unit 1:

Formal notion of sets, Number Sets, Abstract Sets.

#### Unit 2:

Operations on Sets, Union, Intersection complementation, De'Morgan Laws Cardinal Arithmetic

#### Unit 3:

Relations, sets as Functions, Countability, The notion of infinite cardinality

#### Unit 4:

Applications of Sets to Analytic Geometry Graphs of functions, tracing of Curves

#### Unit 5:

Applications to Quantitative Sciences

#### REFERENCES

1. Courant.R, Robbins ,What is Mathematics. Oxford University Press.
2. Kalyan Sinha, Rajeeva Karandikar, C.Musili and others, Understanding Mathematics, University Press.



### 3.3 Differential Geometry

#### Objective:

The student familiar with analytical geometry would find this course as an advancement of calculus methods to study functions defined on generalized spaces. In particular Euclidean geometry is generalized to define notions like frame fields, shape operators, Curvature etc.

#### Unit 1:

Euclidean spaces, tangent vectors to them, vector fields, directional derivatives, curves in  $E^3$ . 1-forms, differential forms, mappings on Euclidean spaces, derivative map, dot product in  $E^3$ , frame fields.

#### Unit 2:

Cross product of tangent vectors, curves in  $E^3$ , arc length, reparametrisation, Frenet formulas, Frenet frame field, curvature, torsion and bitorsion of a unit speed curve.

#### Unit 3:

Arbitrary speed curves, Frenet formulas for arbitrary speed curves, covariant derivatives, Frame field in  $E^3$ , connection forms of a frame field, Cartan's structural equations.

#### Unit 4:

Calculus on a surface, co-ordinate patch, proper patch, surfaces in  $E^3$ , Monge patch, examples, differentiable functions and tangent and normal vector fields on a surface. Mapping of surfaces, topological properties of surfaces, Manifolds.

#### Unit 5:

Shape operators, Normal curvature, Gaussian curvature, computational techniques special curves in surfaces.

#### REFERENCES:

1. Barrett O. Neill, Elementary Differential Geometry, Academic Press, New York (1998)
2. Andrew Priestly, Differential Geometry, Springer
3. Nirmala Prakash, Differential Geometry an Integral approach, Tata McGraw Hill, New Delhi (2001)
4. T.J. Willmore, An introduction to Differential Geometry, Oxford University Press (1999)
5. S. Kumaresan, Differential Geometry and Lie Groups, TRIM Series, HBA

### 3.4 Numerical Analysis

#### Unit 1:

Linear system of Equations: Direct methods- Gauss elimination, Gauss – Jordan, LU – factorization. Iterative methods: Jacobi's, Gauss – Seidel, SOR methods, convergence criteria. Eigen Values and Eigen vectors for symmetric matrices.

#### Unit 2:

Numerical Solution of Ordinary and Partial differential Equations: Introduction, Single/Multi step methods – Picard's theorem, Euler's methods, Modified Euler's method, Derivation of Rung-Kutta 2<sup>nd</sup> order method, Runge – Kutta 4<sup>th</sup> order method, Error analysis, Classification of P.D.E., Finite difference approximations to derivatives, shooting method.

#### Unit 3:

Parabolic P.D.E: One –dimensional heat equation, Explicit and implicit finite difference scheme, Thomas algorithm. Crank-Nicholson method, Gauss-Seidal iterative scheme for Crank-Nicholson method, Successive over relaxation, parabolic equation with derivative boundary condition, ADI method. Parabolic equation in cylindrical & spherical co-ordinates.

#### Unit 4:

Elliptic P.D.E: Laplace equation, Poisson equation, explicit finite difference method, implicit method, Derivative boundary conditions, iterative method.

#### Unit 5:

Hyperbolic P.D.E: Method of characteristics, solution of hyperbolic equation by characteristics, finite difference methods, and explicit finite- difference method, stability of explicit finite- difference method, implicit method.

#### REFERENCES:

1. Jain, lyengar and jain: Numerical methods for scientific and engineering Computation, Wiley Eastern, (1983).
2. Conte s. D. and De Boor, Introduction to Numerical Analysis, McGraw Hill.
3. Hilderband F.B., Introduction to Numerical Analysis, Ed. 5,Tata McGraw Hill, New Delhi,(1986).
4. Carnahan, Luther and Wikes, Applied Numerical Methods. John Weiley(1969)
5. Atkinson K.E, an Introduction to Numerical Analysis,3<sup>rd</sup> Ed, John Weiley and Sons(1989)
6. Smith G. D., Numerical Solution of Partial Differential Equations: Finite Difference Methods, Oxford University press, 2003.



### 3.5 Algebraic Topology

#### Objectives:

After understanding point set topology the students would soon realize that it has its own limitations in classifying certain spaces in terms of topological invariants. Algebraic methods are forced into the realm of topology and geometry to overcome some of the limitations.

#### Unit-1

Review of point set topological concepts, Quotient Topological Spaces, Paths and homotopy, homotopy equivalence, contractibility, deformation retracts.

#### Unit-2

Basic constructions: cones, mapping cones, mapping cylinders, suspension.

#### Unit-3

Cell complexes, subcomplexes, CW pairs. Fundamental groups. Covering spaces, lifting properties, deck transformations. Universal coverings. Examples (including the fundamental group of the circle) and applications (including Fundamental Theorem of Algebra).

#### Unit-4

Brouwer Fixed Point Theorem and Borsuk-Ulam Theorem. Van Kampen's Theorem.

#### Unit-5

Simplicial complexes, barycentric subdivision, stars and links, simplicial approximation. Simplicial Homology. Singular Homology. Mayer-Vietoris Sequences. Long exact sequence of pairs and triples. Homotopy invariance and excision.

#### REFERENCES:

1. Anant R. Shastry, Basic Algebraic Topology, CRC press, 2013.
2. Hatcher, Algebraic Topology, Cambridge Univ. Press, Cambridge, 2002
3. Edwin H. Spenier, Algebraic Topology, Springer, 1994.
4. James R. Munkres, Elements of Algebraic Topology, Westview Press, 1996.
5. W. Massey, A Basic Course in Algebraic Topology, Springer-Verlag, Berlin, 1991.
6. J.R. Munkres, Elements of Algebraic Topology, Addison Wesley, 1984.
7. J.J. Rotman, An Introduction to Algebraic Topology, Springer (India), 2004.
8. H. Seifert and W. Threlfall, A Textbook of Topology, translated by M. A. Goldman, Academic Press, 1980.
9. J.W. Vick, Homology Theory, Springer- Verlag, 1994.
10. James Dugundji, TOPOLOGY, PHI, 2000.
11. B.V.Limaye and Lahiri, Introduction to Algebraic Topology, Narosa Publication.
12. M.J. Greenberg and J. R. Harper, Algebraic Topology, Benjamin, 1981.
13. W. Fulton, Algebraic topology: A First Course, Springer-Verlag, 1995.



### 3.6 OEC

#### OEC – I STATISTICS AND QUANTITATIVE TECHNIQUES

##### Objectives:

This Course would enable the student to get acquainted with the statistical tools in order to handle massive data and arrive at conclusions. Mathematical aptitude and thinking is crucially used while dealing with quantitative problems.

##### Unit-1

Basic Ideas in Statistics, Function of Statistics, Methods of data collection, representation of data, Frequency distribution, Diagrammatic and graphical representations.

##### Unit-2

Central tendencies, Arithmetic Mean, Weighted arithmetic mean, median, mode, Geometric mean, Harmonic mean, Merits, Demerits.

##### Unit-3

Moment, Skewness, Kurtosis, Applications

##### Unit-4

Simple Regression, Multiple Regression, Correlation.

##### Unit-5

Time Series, meaning and Utility, Components of Time Series, Additive and Multiplicative Models, Methods of estimating trends, (Linear and exponential fitting), Autoregressive Models.

##### REFERENCES:

1. Das.M.J, Statistical Methods, Das and Co Publishers Kolkata
2. Miller,J.E.Freud, Mathaematical Statistics with applications,Pearson, New Delhi.
3. Gupta and Gupta, Business Statistics, Sultann Chad Publishers
4. Chandan.J, Statistics for Business Economics, Vikas Publishers

## SEMESTER – IV

### 4.1 Functional Analysis

#### Objectives:

This course on functional analysis is introductory. The student is introduced to Banach spaces and operators defined on them. Similarly Hilbert spaces are studied basically to drive home a point that geometry is understood better by means of an inner product.

#### Unit-1

Functional Analysis Norm on a linear space over  $F$  (either  $R$  or  $C$ ), Banach space, Examples. Norm on Quotient space, Continuous Linear Transformation of normed linear space. The Banach space  $B(N, N')$  for Banach spaces  $N, N'$ .

#### Unit-2

Dual space of normed linear space, Equivalence of norms, Dual space of  $C[a, b]$ , Isometric isomorphism.

#### Unit-3

Hahn-Banach theorem and its applications, Separable normed linear space

#### Unit-4

Canonical embedding of  $N$  into  $N^{**}$ . Reflexive spaces, Open mapping theorem, Closed graph theorem, Principle of Uniform boundedness (Banach-Steinhaus Theorem), Projection on Banach spaces. Hilbert spaces, Definition and examples, Orthogonal complements, Orthonormal basis, Gram-Schmidt process of orthonormalisation, Bessel's inequality, Riesz Fisher Theorem.

#### Unit-5

Adjoint of an operator, Self adjoint, normal, unitary and projection operators.

#### REFERENCES:

1. G.F. Simmons: Introduction to Topology and Modern Analysis, McGraw Hill Book company Inc (1962)
2. C. Goffman and G. Pedrick: First Course in Functional Analysis, Prentice Hall of India Pvt Ltd N. Delhi (1974)
3. B.V. Limaye: Functional Analysis 2nd Edition, New Age International (P) Ltd Publication 1997.
4. D. Somasundaram, Functional Analysis, S. Vishwanathan Printers and Publishers Pvt, Limited (1994)
5. Ponnuswamy, Foundations of Functional analysis, Narosa.
6. K. Chandrashekara Rao, Functional Analysis, Narosa



## 4.2 Probability Theory

### Objective:

The course shall provide the basic aspects of probability to deal with stochastic phenomena. Understanding of uncertainties mathematically is indeed a daunting task and the statistical features are very well studied by probabilistic arguments.

### Unit 1

Random Experiments, Sample spaces, Sets, Events, Algebras, Elements of combinatorial analysis, Classical definition and calculation of Probability, Independence of events

### Unit 2

Random variables, Distribution functions, Moments, Probability and moment generating functions, Independence of random variables, Theoretical distributions: Binomial, Poisson and Normal distribution and their properties.

### Unit 3

Correlation and Regression: Definition meaning scatter diagram method, Karl Pearson's method, Probable error, Standard error and Rank correlation. Regression: Definition: meaning two lines of regression, regression coefficients, standard error and relation between correlation and regression.

### Unit 4

Introduction to various discrete and continuous random variables, Limiting distributions of some random variables Distributions of functions of random variables, Bi-variate distributions, Conditional and marginal distributions, Conditional expectation and variance, Co-variance and correlation co-efficient.

### Unit 5

Elementary understanding of data: Frequency curves, Empirical measures of location, spread, empirical moments, Analysis of Bi-variate data, fitting of distributions.

### REFERENCES

1. Siva Athreya, V.S. Sunder, Measure and Probability, CRC Press
2. William Feller, Introduction to Probability and its applications, Vol - I, 3rd Edition
3. F.M.Dekking, C. Kraaikamp and others, A Modern introduction to Probability and Statistics, Springer Publication.
4. Athanasios Papoulies, Unnikrishna Pillai, Probability, random Variables and Stochastic Processes, Tata McGraw Hill.
5. K.B.Athreya, B.K.Lahiri, Measure Theory and Probability Theory, Hindustan BookAgency, TRIM Series.



## 4.4 Elective

### Elective – I Galois Theory

#### Objectives:

Finding solutions in closed form for a quintic by radicals is an impossible task and this fact was proved by 'Galois' through his higher algebra. Such studies lead to group theoretic techniques to understand equations and their solutions. This course provides an introductory account of such studies.

#### Unit 1

Field extensions, Characteristic of a Field, Finite fields, splitting field of a polynomial.

#### Unit 2

Algebraic extensions, Algebraic closure, algebraically closed field, Separable Extension, Simple extension, Primitive element theorem.

#### Unit 3

Inseparable extension, Purely inseparable extension, Perfect field, Imperfect field, Normal Extension Group of automorphisms of Field extensions.

#### Unit 4

Linear independence of characters, Artin's Theorem, Norm and Trace, Cyclic extension, Hilbert Theorem 90, Artin-Schreier Theorem

#### Unit 5

Solvable extension, Solvability by Radicals, Insolvability of the Quintic, Theorem of Abel – Ruffini. Galois groups of quadratic, cubic and quartic polynomials over the rational field

#### REFERENCES:

1. J.J.Rotman, Galois Theory, Univeritext, Springer 1990.
2. D.J.H.Garling, A Course in Galois Theory CUP, 1986
3. Ian Stewart, Galois Theory, Chapman and Hall, London, New York.
4. I.N.Herstein, Topics in Algebra Blaisidel, NY.
5. Sulrjeet Singh and Quazi Zameerudin, Modern Algebra. Vikas Publications.

## Elective – II Number Theory and Cryptography

### Objectives:

Students of this course will get a firsthand account of number theory techniques which can solve problems in cryptology. Students will be studying various ciphers which involve structures and techniques of number theory.

### Unit 1

Divisibility and Euclidean algorithm, Congruences and their applications to factoring.

### Unit 2

Finite Fields, Legendre symbol, quadratic reciprocity, Jacobi symbol.

### Unit 3

Cryptosystems, Digraph Transformations and enciphering matrices, RSA cryptosystem.

### Unit 4

Primality and factoring, Pseudoprimes, Carmichael numbers, Primality tests, Strong Pseudoprimes, Montecarlo method, Fermat factorization, Factor base, implication for RSA, continued fraction method.

### Unit 5

Elliptic curves, Basic facts, elliptic curves over  $\mathbb{R}, \mathbb{Q}, \mathbb{C}$  and finite fields, Hasse Theorem, Weil Conjectures (without proof), elliptic curve cryptosystem.

### REFERENCES

1. N.Koblitz, A course in Number theory and Cryptology, GTM Springer 1987.
2. Rosen.M, Ireland K, A Classical introduction to Number Theory, Spinger.
3. David.Bressoud, Factorization and Primality testing, UTM, Springer 1989



## Elective – III Fluid Mechanics

### Objectives:

This course offers classical problems of fluid mechanics and their solutions thus giving a good account of advanced calculus methods for real world problems.

#### Unit 1

Streamlines and Paths of particles, Bernoulli Equations, Bernoulli's Theorem, Equations of motion, Equation of continuity, Boundary conditions, Irrotational motion, Kelvin's Minimum energy theorem. 2-Dimensional motion, Stream function, Rankine's method, Velocity potential.

#### Unit 2

Streaming motions: Complex potential, circulation, Circle theorem, Joukowski transformation, Theorem of Blasius, Aerofoil Joukowski's Hypothesis, The Theorem of Kutta and Joukowski, Lift on a aerofoil.

#### Unit 3

Sources and sinks, 2-dimensional source, combination of sources and stream, doublet, the method of images, Image of a doublet in a sphere, source outside a cylinder, source in compressible flow.

#### Unit 4

Stoke's stream function, Axisymmetric motion, Butler's sphere theorem, Image of a source in a sphere, force on an obstacle, spheres and ellipsoids, circle harmonics, Kelvin's inversion theorem. Weiss Sphere Theorem

#### Unit 5

The equation of motion, Boundary conditions in viscous flows, Flow through a pipe, Axisymmetric motion, drag on a slowly moving sphere.

### REFERENCES:

1. Charlton, Fluid Dynamics
2. D. E. Rutherford, Fluid Mechanics
3. J. L. Bansal, Viscous Fluid Dynamics.
4. G. K Bachelor -.An Introduction to Fluid dynamics.
5. Landau, Lifchitz, fluid Mechanics.

## Elective – IV Commutative Algebra

### Objectives:

This course is an extension of the familiar abstract algebra course . However the emphasis here is on the commutative set-up which has linkages to algebraic geometry and function algebras.

### Unit 1

Rings, Subrings, ideals, quotient rings, Definitions and examples. Ring homomorphism, isomorphism theorems Correspondence theorem . Zero-divisors, nilpotent elements and units in a ring. Prime ideal ,Maximal ideal. Nilradical abd Jacobson radical of a ring. Operations on ideals. Extensions and contractions of ideals. Polynomial rings. Poweer series ring.

### Unit 2

Modules, submodules, quotient modules, Definition and examples. Homomorphisms of modules. Isomorphism theorems Correspondence theorem. Operations on submodules. Direct product and direct sum of modules. Finitely generated modules. Nakayama lemma.

### Unit 3

Rings and modules of fractions. Local properties . Extended and contracted ideals in rings and fractions.

### Unit 4

Noetherian module, Artinian module. Composition series of a module. Modules of finite length . Jordan –Holder theorem.

### Unit 5

Noetherian ring Artinian ring. Hilbert basis theorem. Hilbert Nullstellensatz. Algebraic geometry Connections

### REFERENCES :

1. M.F.Atiyah and I.G.Macdonald , Introduction to Commutative Algebra, Addison-Wesley Publishing Company, 1969 .
2. C.Musili, Introduction to Rings and Moduls , Narosa Publishing House , Second Revised Edition, 1994.
3. N.S.Gopalkrishnan, Commutative Algebra , Oxonioan Press Private Limited, New Delhi (1984)
4. O.Zariski and P.Samuel , Commutative Algebra, Vol I Van NostrandCompan



## Elective - V Mathematical Physics

### Objectives:

The students are exposed to the powerful mathematical techniques required to quantify the experimental data. Integral equations, Laplace, Fourier and other integral transforms help in achieving the above said goal.

### Unit -1

Integral Transforms: General definition of Integral transforms, Kernels, etc. Development of Fourier integral, Fourier transforms – inversion, Illustration on the use of integral transforms, Laplace, Fourier, Hankel and Mellin transforms to solve ODEs and PDEs - typical examples. Discrete orthogonality and Discrete Fourier transform.

### Unit -2

Integral Equations: Definition, Volterra and Fredholm integral equations. Solution by separable kernel, Neumann's series resolvent kernel and transform methods,

### Unit -3

Convergence for Fredholm and Volterra types. Reduction of IVPs BVPs and eigenvalue problems to integral equations. Hilbert Schmidt theorem, Raleigh Ritz and Galerkin methods.

### Unit -4

Regular and singular perturbation methods: Parameter and co-ordinate perturbations. Regular perturbation solution of first and second order differential equations involving constant and variable coefficients. Include Duffings equation, Vanderpol oscillator, small Reynolds number flow.

### Unit -5

Singular perturbation problems, Matched asymptotic expansions, simple examples. Linear equation with variable coefficients and nonlinear BVP's. Problems involving Boundary layers. Poincare – Lindstedt method periodic solution. WKB method, turning points, zeroth order Bessel function for large arguments, solution about irregular singular points.

### REFERENCE BOOKS

1. I.N. Sneddon – The use of Integral Transforms, Tata Mc Graw Hill, Publishing Company Ltd, New Delhi, 1974
2. R.P. Kanwal: Linear integral equations theory and techniques, Academic Press, New York, 1971
3. C.M. Bender and S.A. Orszag – Advanced mathematical methods for scientists and engineers, Mc Graw Hill, New York, 1978
4. H.T. Davis – Introduction to nonlinear differential and integral equations, Dover Publications, 1962.
5. A.H. Nayfeh – Perturbation Methods, John Wiley & sons New York, 1973
6. Don Hong, J. Wang and R. Gardner. Real analysis with introduction to wavelets and applications, Academic Press Elsevier (2006)

## Elective – VI Coding Theory

### Objectives:

This course introduces the mathematical basis of a successful communication infrastructure involving error correcting codes. Shannon laid down the mathematical framework. The practical problems involved in data communication is solved by the algebraic methods involving Coding theory.

### Unit 1

Preliminaries of Communication Channels and coding requirements, Block Codes, Linear Codes and Hamming Codes.

### Unit 2

Hadamard Codes and generalizations. Constructing Codes from other codes, Reed Muller Codes.

### Unit 3

Bounds on Codes Gilbert Varshamov bounds, Upper bounds, Linear programming bound. Generator Matrix and check polynomials in case of cyclic codes. BCH Codes and Reed - Solomon Codes, Quadratic Residue codes.

### Unit 4

Codes over  $Z_4$ , Quaternary codes, Binary Codes derived Codes over  $Z_4$ , Goppa codes, Minimum distance and generalized BCH Codes.

### Unit 5

Algebraic geometry Codes. Codes arising from Algebraic curves. Statement of Riemann Roch Theorem, applications.

### REFERENCES:

1. J.H. Van Lint, Introduction to Coding Theory, GTM Springer Verlag.
2. W.C. Huffman, Vera Press, Fundamentals of Error Correcting Codes, CUP.



## 4.5 Elective

### **Elective – I Mathematical Finance**

#### **Objectives:**

Modeling Equity market and Derivatives of the financial markets has been mainly through the mathematical finance theory. In this course we trace back its origin and its gradual development with the usage of Probability theory. The initial seeds sown by L.Bachelier and subsequent rigour given by Kolmogorov, Wiener and Ito (through his Ito calculus) and Black-Scholes theory shall be introduced.

#### **Unit 1:**

Mathematics of Financial Markets. Stocks and their Derivatives, Pricing Futures Contracts, Bond Markets, Computing Rate of Return, Interest Rates and Forward Interest Rates. Yield Curves.

#### **Unit 2:**

Methods of Hedging a Stock or Portfolio, Hedging with Puts, Hedging with Collars, Correlation based hedges. Volatility computations, Delta hedging.

#### **Unit 3:**

Interest Rates and Forward Rates, Zero coupon Bonds, Forward Rates and Zero Coupon Bonds. Computations based on  $Y(t)$ ,  $P(t)$ , Swaps and related arbitrage. Pricing and hedging a Swap. Arithmetic and Geometric Interest rates. Interest Rate Models in discrete and continuum setting., Bond price dynamics.

#### **Unit 4:**

Binomial Trees, Expected Value Pricing, Arbitrage, Pricing probability Binomial Model for Pricing Options, N-Period Binomial model for Hedging.

#### **Unit 5:**

A continuous time stock Model, Ito Calculus and Stochastic Models, The discrete Model, Black Scholes formula, Put Call Parity, Trees and continuous Models. Sensitivity issues.

#### **REFERENCES**

1. Oksendal, Stochastic Differential Equations. Springer
2. Williams R.J, Introduction to Mathematics of Finance, Universities Press
3. V.Goodman, J.Stampfli, Mathematics of Finance, Thomson Brooks/Cole, 2001.
4. S.Ross, Mathematical Finance, CUP.
5. J.C.Hull, Options, Futures and Other Derivatives, Pearson Publication
6. S.Shreve, Stochastic Calculus and applications, Springer.

## Elective – II Operations Research

### Objectives:

Optimization problems are studied through this course. Such problems are formulated to reach certain targets in the industrial and military applications. The course introduces to the students the techniques involved in achieving optimal goals in a given problem.

### Unit-I

The linear programming problem, properties of a solution to the linear programming problem, generating extreme point solution, simplex computational procedure, development of minimum feasible solution, the artificial basis techniques, a first feasible solution using slack variables, two phase and Big-M method with artificial variables.

### Unit-II

General Primal-Dual pair, formulating a dual problem, primal-dual pair in matrix form, Duality theorems, complementary slackness theorem, duality and simplex method, economic interpretation of duality, dual simplex method.

### Unit-III

General transportation problem, transportation table, duality in transportation problem, loops in transportation tables, LP formulation, solution of transportation problem, test for optimality, degeneracy, transportation algorithm (MODI method), time- minimization transportation problem.

### Unit-IV

Mathematical formulation of assignment problem, assignment method, typical assignment problem, the traveling salesman problem. Game Theory: Two-person zero-sum games, maximin minimax principle, games without saddle points (Mixed strategies), graphical solution of  $2 \times n$  and  $m \times 2$  games, dominance property, arithmetic method of  $n \times n$  games, general solution of  $m \times n$  rectangular games.

### Unit-V

Integer Programming: Gomory's all I.P.P. method, constructions of Gomory's constraints, Fractional cut method-all integer and mixed integer, Branch-and-Bound method, applications of integer programming. Dynamic Programming: The recursive equation approach, characteristics of dynamic programming, dynamic programming algorithm, solution of-Discrete D.P.P., some applications, solution of L.P.P. by Dynamic Programming.

### REFERENCES

1. Taha, Operations Research, Pearson Education; Eighth edition (2011)
2. Kambo N. S., Mathematical Programming,
3. G. Hadley, Linear Programming, Addison Wesley.
4. Gass, S. L., Linear Programming, Courier Dover Publications, 2003



## Elective – III Graph Theory

### Objectives:

This course highlights the simplest discrete structure involved in a set and provides a good account of combinatorial notions to characterize certain invariants arising from a graph. Applications in engineering, social sciences and elsewhere are plenty and students can get a hands on experience in modeling several real world problems.

### Unit 1

Factorization-1-factorization, 2-factorization, decomposition and labellings of Graphs.

### Unit-2

Coverings, Vertex covering, Edge covering, Independence number, Matchings and Matching polynomials.

### Unit-3

Planarity, Planar Graphs, outer planar graphs, Kuratowski criterion for planarity and Eulers formula, Graph valued functions: Line graphs, subdivision graphs and total graphs

### Unit-4

Colorings, Chromatic numbers and chromatic polynomials, Spectra of Graphs: Adjacency Matrix, Incidence Matrix, Characteristic polynomial, Eigen values, Graph parameters, Strongly regular graphs, and Friendship theorem

### Unit-5

Groups and Graphs, Automorphism group of Graph. Operations on Permutation graphs, The Group of Composite graphs, Domination: Dominating sets, Domination numbers, Domatic number and its bounds, independent domination of a number of a Graph, Other domination parameters. Theory of external graphs and Ramsey theory.

### REFERENCES:

1. M.Behzad, G.Charatrand and L.Lesniak-Foster: Graphs and Digraphs, Wadsworth,Belmont,Calif (1981)
2. Narsing Deo; Graph theory with Applications to Engineering and Computer Science, Prentice Hall INDIA, 1995.
3. J.A.Bondy and V.S.R.Murthy, Graph Theory with applications McMillan, London.
4. F.Buckley and F.Harary: Distance in Graphs, Addison Wesley, 1990.
5. Diestel: Graph Theory, Springer Verlag, Berlin
6. R.Gould: Graph Theory, Benjamin Cummins Publication Company Inc, Calif 1998.
7. F.Harary Graph Theory, Addison Wesley, Reading Mass 1969.
8. O.Ore: Theory of Graphs, Amer.Math, Soc.College Publications-38 Providence 1962.
9. D.Cvetkovic, M.Doob and H.Sachs, Spectra in Graphs, Academic Press, 1980.
10. Tulasiraman and M.N.S.Swamy, Graphs Networks and Algorithms, John Wiley (1989)

## Elective – IV Fourier Analysis

### Unit 1

Basic Properties of Fourier Series: Uniqueness of Fourier Series, Convolutions, Cesaro and Abel Summability, Fejer's theorem, Poisson Kernel and Dirichlet problem in the unit disc. Mean square Convergence, Example of Continuous functions with divergent Fourier series.

### Unit 2

Distributions and Fourier Transforms: Calculus of Distributions, Schwartz class of rapidly decreasing functions, Fourier transforms of rapidly decreasing functions, Riemann Lebesgue lemma, Fourier Inversion Theorem, Fourier transforms of Gaussians.

### Unit 3

Tempered Distributions: Fourier transforms of tempered distributions, Convolutions, Applications to PDEs (Laplace, Heat and Wave Equations), Schrodinger-Equation and Uncertainty principle.

### Unit 4

Paley-Wiener Theorems, Poisson Summation Formula,

### Unit 5

Radial Fourier transforms and Bessel's functions. Hermite functions. Wavelets and X-ray tomography. Applications to Number Theory.

### References:

1. R. Strichartz, A Guide to Distributions and Fourier Transforms, CRC Press.
2. E.M. Stein and R. Shakarchi, Fourier Analysis: An Introduction, Princeton University Press, Princeton 2003.
3. Richards and H. Youn, Theory of Distributions and Non-technical Approach, Cambridge University Press, Cambridge, 1990.



## Elective – V Banach Algebra

### Objectives:

To understand the algebra of infinite dimensional function spaces Banach algebras are naturally studied. Analysis is extended to the functional algebras while studying certain extremal formulas.

### Unit 1

Preliminaries, Banach spaces, Weak topologies on Banach spaces, Banach valued functions and their derivatives, Holomorphic functions, Banach space valued measures and Integration.

### Unit 2

Definition of Banach Algebra, Homomorphisms, Spectrum, Basic properties of Spectra, Gelfand- Mazur Theorem, Spectral Mapping Theorem, group of invertible elements.

### Unit 3

Ideals, Maximal Ideals and Homomorphisms, Semisimple Banach Algebras

### Unit 4

Gelfand Topology, Gelfand Transform, Involutions, Banach-C\*-Algebras, Gelfand Naimark Theorem, Applications to Non-Commutative Banach Algebras, Positive functions.

### Unit 5

Operators on Hilbert Spaces, Commutativity theorem, Resolution of the identity spectral theorem, A Characterization of Banach C\*-Algebras

### REFERENCES

1. Rudin.W, Functional Analysis.
2. Bachman and Narice L, Functional Analysis , Academic Press.
3. B.V.Limaye, Functional Analysis, New Age International Limited
4. S.K.Berbenon, Lectures in Functional Analysis and Operator Theory, Narosa, 1979.

## Elective – VI Mathematical Modeling

### Unit 1

Some basic topics in Nonlinear Waves: Shock waves and hydraulic jumps. Description and various physical set ups where they occur: traffic flow, shallow water.

### Unit 2

Fundamental concepts in continuous applied mathematics. Continuum limit. Conservation laws, quasi-equilibrium. Kinematic waves. Traffic flow (TF). Continuum hypothesis. Conservation and derivation of the mathematical model.

### Unit 3

Integral and differential forms. Other examples of systems where conservation is used to derive the model equations (in nonlinear elasticity, fluids, etc.), Linearization of equations of TF and solution. Meaning and interpretation. Solution of the fully nonlinear TF problem.

### Unit 4

Method of characteristics, graphical interpretation of the solution, wave breaking. Weak discontinuities, shock waves and rarefaction fans. Envelope of characteristics. Irreversibility in the model.

### Unit 5

Quasilinear First Order PDE's, Shock structure, diffusivity. Burger's equation. The Cole-Hopf transformation. The heat equation: derivation, solution, and application to the Burger's equation. Inviscid limit and Laplace's method.

### References:

1. R. Haberman, Mathematical Models, Mechanical Vibrations, Population Dynamics and Traffic flow, SIAM.
2. C. C. Lin and L. Segal, Mathematics Applied to Deterministic Problems in the Natural Sciences, SIAM.
3. F. Y. M. Wan, Mathematical Models and their Analysis, Harper and Row.
4. J. D. Logan, An Introduction to Nonlinear Partial Differential Equations, J. Wiley.
5. R. D. Richtmyer and K. W. Morton Difference Methods for Initial-Value Problems, Interscience, Wiley, Krieger.
6. C. Fowler, Mathematical Models in the Applied Sciences, Cambridge U. Press.
7. J. J. Stoker, Nonlinear Vibrations in Mechanical and Electrical Systems, J. Wiley.
8. G. B. Whitham, Linear and Nonlinear Waves, J. Wiley.
9. R. Haberman, Applied Partial Differential Equations With Fourier Series and Boundary Value Problems, Prentice Hall.