SEMESTER – III

3.1 MEASURE THEORY & LEBESGUE INTEGRATION

Unit 1:

Lebesgue outer measure, Lebesgue measurable sets and measurable functions.

Unit 2 :

Algebra of measurable functions . Egoroff's theorem. Lebesgue integral of bounded function over a set of finite measure.

Unit 3:

Bounded convergence theorem. Fatou's lemma. General Lebesgue integral. Lebesgue's monotone convergence theorem.

Unit 4:

Lebesgue General (Dominated) convergence theorem. Differential of an integral Lp space. Completeness of Lp-space.

Unit 5:

Product Measure, Fubini theorems, Radon-Nikodym theorem.

REFERENCES:

1. H.L.Royden: Real Analysis (Chapter 1,3,4,5 and 6).3rd Edition,MacMillan,NewYork(1963)
2. Inder Kumar Rana, Measure Theory and Integration, Narosa.
3. C.Goffman : Real Functions,Holt,Rinehart and Winston Inc.New York (1953)
4. P.K.Jain and V.P.Gupta : Lebesgue Measure and Integration, Wiley Eastern Ltd.(1986)
5. P.Halmos, Measure Theory, Narosa Publishers.

3.2 DIFFERENTIAL GEOMETRY

Unit 1:

Euclidean spaces, tangent vectors to them, vector fields, directional derivatives, curves in E_3. 1- forms, differential forms, mappings on Euclidean spaces, derivative map, dot product in E_3, frame fields.

Unit 2:

Cross product of tangent vectors, curves in E_3, arc length, reparametrisation, Frennet formulas, Frenet frame field, curvature, torsion and bitorsion of a unit speed curve.

Unit 3:

Arbitrary speed curves, Frenet formulas for arbitrary speed curves, covariant derivatives, Frame field in E_3, connection forms of a frame field, Carton's structural equations.

Unit 4:

Calculus on a surface, co-ordinate patch, proper patch, surfaces in E_3, Monge patch, examples, differentiable functions and tangent and normal vector fields on a surface. Mapping of surfaces, topological properties of surfaces, Manifolds.

Unit 5:

Shape operators, Normal curvature, Gaussian curvature, computational techniques special curves in surfaces.

REFERENCES:

1. Barrett O.Neill,Elementary Differential Geometry,Academic Press,New York(1998)
2. Andrev Priestly , Differential Geometry, Springer
3. Nirmala Prakash,Differential Geometry an Integral approach ,Tata McGraw Hill,New Delhi (2001)
4. T.J.Willmore,An introduction to Differential Geometry, Oxford University Press(!999)
5. S.Kumaresan, Differential Geometry and Lie Groups, TRIM Series, HBA

3.3. NUMERICAL ANALYSIS

Unit -1

Criterion -Aitken's Δ 2- process -Sturm sequence method to identify the number of real roots – Newton - Raphson's methods convergence criterion Ramanujan's Method - Birge-Vieta method, and Bairstow method

Unit-2

Linear and Nonlinear system of Equations: Gauss Eliminations with Pivotal Strategy. LU - decomposition methods – Crout's, Cholesky method, Partition method – Jacobi and Gauss Seidel Itervative Methods with convergence criterion consistency and ill conditioned system of equations.

Unit-3

Iterative methods for Nonlinear system of equations, Fixed point iteration method, Newton Raphson, Quasi Newton and Successive Over Relaxation methods for Nonlinear system of Equations. Tri-diagonal system of equations –Thomas Algorithm. Eigen values and eigenvector of symmetric matrix.

Unit -4

Interpolation: Lagrange, Hermite, Cubic-spline's (Natural, Not a Knot and Clamped) - with uniqueness and error term, for polynomial interpolation. Bivariate interpolation.
Orthogonal polynomials Grams Schmidth Orthogoralization procedure and least square, Chebyshev and Rational function approximation.

Unit-5

Numerical differentiation and Integration: Method based on interpolation, Gaussian quadrature, Gauss-Legendre, Gauss-Chebeshev formulas, Gauss Leguree, Gauss Hermite and Spline intergation– Integration over rectangular and general quadrilateral areas and multiple integration with variable limits.

TEXT BOOKS

1. M. K. Jain, S. R. K.Iyengar and R.K.Jain : Numerical methods for scientific and engineering computation, Wiley Eastern Ltd. 1993, Third Edition.
2. C.F.Gerald, and P.O. Wheatley : Applied Numerical Methods, Low- priced edition, Pearson Education Asia 2002, Sixth Edition.
3. D.V. Griffiths and I.M. Smith, Numerical Methods for Engineers, Blackwell Scientific Publications (19991).

REFERENCE BOOKS

1. S.C.Chapra, and P.C. Raymond : Numerical Methods for Engineers, Tata Mc Graw Hill, New Delhi, 2000
2. R.L.Burden, and J.Douglas Faires : Numerical Analysis, P.W.S. Kent publishing Company, Boston ,1989 Fourth edition.
3. S.S. Sastry : Introductory methods of Numerical analysis, Prentice- Hall of India, New Delhi, 1998.
4. Paruiz Moin: Fundamentals of Engineering Numerical analysis, Cambridge University Press (2006)
5. K. K. Mishra, A Handbook on Numerical technique Lab, MATLAB based

3.4. ELECTIVE

I -MATHEMATICAL FINANCE

Unit 1:

Mathematics of Financial Markets. Stocks and their Derivatives, Pricing Futures Contracts, Bond Markets, Computing Rate of Return, Interest Rates and Forward Interest Rates. Yield Curves.

Unit 2:

Methods of Hedging a Stock or Portfolio, Hedging with Puts, Hedging with Collars, Correlation based hedges. Volatility computations, Delta hedging.

Unit 3:

Interest Rates and Forward Rates, Zero coupon Bonds, Forward Rates and Zero Coupon Bonds. Computations based on Y(t), P(t),Swaps and related arbitrage. Pricing and hedging a Swap. Arithmetic and Geometric Interest rates. Interest Rate Models in discrete and continuum setting., Bond price dynamics.

Unit 4:

Binomial Trees, Expected Value Pricing, Arbitrage, Pricing probability Binomial Model for Pricing Options, N-Period Binomial model for Hedging.

Unit 5:

A continuous time stock Model, Ito Calculus and Stochastic Models, The discrete Model, Black Scholes formula, Put Call Parity, Trees and continuous Models. Sensitivity issues.

REFERENCES

1. Oksendal, Stochastic Differential Equations.Springer
2. Williams R.J, Introduction to Mathematics of Finance, Universities Press
3. V.Goodman, J.Stampfli, Mathematics of Finance, Thomson Brooks/Cole,2001.
4. S.Ross, Mathematical Finance, CUP.
5. J.C.Hull, Options, Futures and Other Derivatives, Pearson Publication
6. S.Shreve, Stochastic Calculus and applications, Springer.

3.4. ELECTIVE

II - FLUID MECHANICS

Unit 1

Kinematics of fluids in motion; Velocity of a fluid at a point, Stream lines. Path lines and Streak lines. Velocity potential. Vorticity vector, local and particle rate of change, equation of Continuity. Motion of inviscid fluids ; Euler's Equations of motion. Bernoulli's equation. Equation of motion by flux method.

Unit2

Motion of inviscid fluids:- Steady motion under conservative body forces, Potential theorems, - Kelvin's theorem – Impulsive motion - Dimensional analysis – Non-dimensional numbers.

Unit3

Two dimensional flows of inviscid fluids:- Meaning of two-dimensional flow - Stream function – Complex potential - Line sources and sinks - Line doublets and vortices - Images - Milne-Thomson circle theorem and applications - Blasius theorem and applications.

Unit 4

Motion of Viscous fluids:- Stress tensor – Navier-Stokes equation - Energy equation - Simple exact solutions of Navier-Stokes equation: (i) Plane Poiseuille and Hagen-Poiseuille flows (ii) Generalized plane Couette flow (iii) Steady flow between two rotating concentric circular cylinders (iv) Stokes's first and second problems (vi) Slow and steady flow past a rigid sphere and cylinder. Diffusion of vorticity - Energy dissipation due to viscosity.

Unit 5

Boundary layer concept –Derivation of Prandtl boundary layer equations – Boundary layer along flat plate, Blasius solution , Boundary layer on a surface with pressure gradient, Momentum Integral theorem.

REFERENCES:

1. F. Chorlton : Text book of Fluid Dynamics, Van Nostrand, 1967

1. 2. Z. U. A.Warsi : Fluid Dynamics, CRC Press (2nd edition), 1999.

3. J. L. Bansal, Viscous Fluid Dynamics.

4. S. W. Yuan : Foundations of Fluid Mechanics, Prentice Hall, 1976.

5. G. K Bachelor -.An Introduction to Fluid dynamics.

3.4. ELECTIVE

III-COMMUTATIVE ALGEBRA

Unit 1

Rings, Subrings, ideals, quotient rings, Definitions and examples. Ring homomorphism, isomorphism theorems Correspondence theorem . Zero-divisors, nilpotent elements and units in a ring. Prime ideal ,Maximal ideal. Nilradical abd Jacobson radical of a ring. Operations on ideals. Extensions and contractions of ideals. Polynomial rings. Poweer series ring.

Unit 2

Modules, submodules, quotient modules, Definition and examples. Homomorphisms of modules. Isomorphism theorems Correspondence theorem. Operations on submodules. Direct product and direct sum of modules. Finitely generated modules. Nakayama lemma.

Unit 3

Rings and modules of fractions. Local properties . Extended and contracted ideals in rings and fractions.

Unit 4

Noetherian module, Artinian module. Composition series of a module. Modules of finite length . Jordan –Holder theorem.

Unit 5

Noetherian ring Artinian ring. Hilbert basis theorem. Hilbert Nullstellensatz. Algebraic geometry Connections.

REFERENCES :

1. M.F.Atiyah and I.G.Macdonald , Introduction to Commutative Algebra, Addison-Wesley Publishing Company, 1969 .
2. C.Musili, Introduction to Rings and Moduls , Narosa Publishing House , Second Revised Edition, 1994.
3. N.S.Gopalkrishnan, Commutative Algebra , Oxonioan Press Private Limited, New Delhi (1984)
4. O.Zariski and P.Samuel , Commutative Algebra, Vol I Van NostrandCompan

3.4. ELECTIVE

IV - CODING THEORY

Unit 1

Preliminaries of Communication Channels and coding requirements, Block Codes, Linear Codes and Hamming Codes.

Unit 2

Hadamard Codes and generalizations. Constructing Codes from other codes,Reed Muller Codes.

Unit 3

Bounds on Codes Gilbert Varshamov bounds, Upper bounds, Linear programming bound. Generator Matrix and check polynomials in case of cyclic codes. BCH Codes and Reed - Solomon Codes, Quadratic Residue codes.

Unit 4

Codes over Z4, Quaternary codes, Binary Codes derived Codes over Z4, Goppa codes, Minimum distance and generalized BCH Codes.

Unit 5

Algebraic geometry Codes. Codes arising from Algebraic curves. Statement of Riemann Roch Theorem, applications.

REFERENCES:

1. J.H. Van Lint, Introduction to Coding Theory, GTM Springer Verlag.
2. W.C. Huffman, Vera Press, Fundamentals of Error Correcting Codes, CUP.

3.5. - ELECTIVE

I. ALGEBRAIC TOPOLOGY

Unit-1

Review of point set topological concepts, Quotient Topological Spaces, Paths and homotopy, homotopy equivalence, contractibility, deformation retracts.

Unit-2

Basic constructions: cones, mapping cones, mapping cylinders, suspension.

Unit-3

Cell complexes, subcomplexes, CW pairs. Fundamental groups. Covering spaces, lifting properties, deck transformations. Universal coverings. Examples (including the fundamental group of the circle) and applications (including Fundamental Theorem of Algebra).

Unit-4

Brouwer Fixed Point Theorem and Borsuk-Ulam Theorem. Van Kampen's Theorem.

Unit-5

Simplicial complexes, barycentric subdivision, stars and links, simplicial approximation. Simplicial Homology. Singular Homology. Mayer-Vietoris Sequences. Long exact sequence of pairs and triples. Homotopy invariance and excision.

REFERENCES:

1. Anant R. Shastry, Basic Algebraic Topology, CRC press, 2013.
2. Hatcher, Algebraic Topology, Cambridge Univ. Press, Cambridge, 2002
3. Edwin H. Spenier, Algebraic Topology, Springer, 1994.
4. James R. Munkres, Elements of Algebraic Topology, Westview Press, 1996.
5. W. Massey, A Basic Course in Algebraic Topology, Springer-Verlag, Berlin, 1991.
6. J.R. Munkres, Elements of Algebraic Topology, Addison Wesley, 1984.
7. J.J. Rotman, An Introduction to Algebraic Topology, Springer (India), 2004.
8. H. Seifert and W. Threlfall, A Textbook of Topology, translated by M. A. Goldman, Academic Press, 1980.
9. J.W. Vick, Homology Theory, Springer- Verlag, 1994.
10. James Dugundji, TOPOLOGY, PHI, 2000.
11. B.V.Limaye and Lahiri, Introduction to Algebraic Topology, Narosa Publication.
12. M.J. Greenberg and J. R. Harper, Algebraic Topology, Benjamin, 1981.
13. W. Fulton, Algebraic topology: A First Course, Springer-Verlag, 1995.

3.5- ELECTIVE

II - NUMBER THEORY AND CRYPTOGRAPHY

Unit 1

Divisibility and Euclidean algorithm, Congruences and their applications to factoring.

Unit 2

Finite Fields, Legendre symbol, quadratic reciprocity, Jacobi symbol.

Unit 3

Cryptosystems, Digraph Transformations and enciphering matrices, RSA cryptosystem.

Unit 4

Primality and factoring, Pseudoprimes, Carmichael numbers, Primality tests, Strong Pseudoprimes, Montecarlo method, Fermat factorization, Factor base, implication for RSA, continued fraction method.

Unit 5

Elliptic curves, Basic facts, elliptic curves over R,Q,C and finite fields, Hasse Theorem, Weil Conjectures (without proof), elliptic curve cryptosystem.

REFERENCES:

1. N.Koblitz, A course in Number theory and Cryptology, GTM Springer 1987.
2. Rosen.M, Ireland K, A Classical introduction to Number Theory, Spinger.
3. David.Bressoud, Facorization and Primality testing, UTM,Springer1989

3.5. ELECTIVE

III - FOURIER ANALYSIS

Unit 1

Basic Properties of Fourier Series: Uniqueness of Fourier Series, Convolutions, Cesaro and Abel Sum ability, Fejer's theorem, Poisson Kernel and Dirichlet problem in the unit disc. Mean square Convergence, Example of Continuous functions with divergent Fourier series.

Unit 2

Distributions and Fourier Transforms: Calculus of Distributions, Schwartz class of rapidly decreasing functions, Fourier transforms of rapidly decreasing functions, Riemann Lebesgue lemma, Fourier Inversion Theorem, Fourier transforms of Gaussians.

Unit 3

Tempered Distributions: Fourier transforms of tempered distributions, Convolutions, Applications to PDEs (Laplace, Heat and Wave Equations), Schrodinger-Equation and Uncertainty principle.

Unit 4

Paley-Wienner Theorems, Poisson Summation Formula,

Unit 5

Radial Fourier transforms and Bessel's functions. Hermite functions. Wavelets and X-ray tomography. Applications to Number Theory.

REFERENCES:

1. R. Strichartz, A Guide to Distributions and Fourier Transforms, CRC Press.
2. E.M. Stein and R. Shakarchi, Fourier Analysis: An Introduction, Princeton University Press, Princeton 2003.
3. Richards and H. Youn, Theory of Distributions and Non-technical Approach, Cambridge University Press, Cambridge, 1990.

3.5. ELECTIVE

IV- FUZZY SETS AND FUZZY SYSTEMS

Unit 1

Introduction, Crisp sets, Fuzzy sets, Significance and Characteristics. Fuzzy sets versus Crisp sets: Properties of alpha cuts, representations of fuzzy sets, extension principle for fuzzy sets.

Unit 2

Types of operations, fuzzy complements, fuzzy intersection, T' – norms fuzzy union, T'-conorms, combination of operations, aggregation of operations,

Unit 3

fuzzy relations, Crisp versus fuzzy sets, Projections and cylindrical extensions, fuzzy equivalence relations, compatibility relations, ordering relations, morphisms. Supremum, infimum fuzzy relations.

Unit 4

Fuzzy measure. Evidence theory, Possibility theory versus Probability theory.

Unit 5

Fuzzy logic, Classical logic, Multivalued logic fuzzy positions, fuzzy quantifiers. linguistic hedges, Inference from conditional fuzzy propositions and qualified propositions. Inference from quantified propositions.

References:

1. George.J.Klir and Bo Yuan, Fuzzy Logic; Theory and Applications,
2. T.J.Ross, Fuzzy Logic with Engineering Applications, Tata McGraw Hill (1997).
3. A. Kaufmann, Introduction to the theory of fuzzy subsets Vol. I, Academic Press (New York) 1975.
4. H.J. Zimmermann, Fuzzy Set Theory and its Applications, Allied Publishers (1991).

3.6. Open Elective Course

I: STATISTICS (Arts and Commerce Stream)

Unit1:

Frequency Distribution, Measure of Central Tendency A.M.G.M., H.M Median,
Mode Standard deviation.

Unit:2

Moments, Moments generation function, Skewness, Correlation

Unit:3

Karls Pearson's Co- efficient of Corrdation, Rank correlation co efficient Regression, line of regression, Equations to the lines of regression Error of prediction.

Unit4:

Probability, Definitions, Addition Law of Probability, Multiplication of law of Probability Baye's theorem.

Unit5:

Binomial Distribution, Mean of binomial distribution, Poisson distribution mean of Poisson distribution Normal distribution, mean of normal distribution.

REFERENCES:

1. Das.M.J, Statistical Methods, Das and Co Publishers Kolkata.
2. Miller,J.E.Freud, Mathaematical Statistics with applications,Pearson, New Delhi.
3. Gupta and Gupta, Business Statistics, Sultann Chad Publishers.
   Chandan.J, Statistics for Business Economics, Vikas Publishers.

3.6. Open Elective Course

II : COMPUTATIONAL METHODS (Science Stream)

Unit-1 :

Solution of algebraic and transcendental equations ; Fixed point iterative method, Bisection method, Regula –Falsi method, Secant method and Newton-Raphson method

Unit-2 :

Linear algebraic system of Equations: Direct method; Gauss Eliminations and Gauss-Jordan methods. Iterative methods ; Jacobi iteration method and Gauss Seidel iteration Method.

Unit 3:

Interpolation: Newton forward and backward interpolation, Lagranges interpolation. Least square approximation (linear, quadratic and cubic).

Unit 4:

Numerical Integration: Trapezoidal Rule, Simpsons 1/3 and 3/8th rule. Numerical solution of derivatives ; Taylor' series method, Euler method and Euler modified method and Runge-kutta 2 and 4th order methods.

Unit 5:

Permutations and Combinations: Introduction, Rules of Sum and Product, Permutations, Combinations, Generation of Permutations and Combinations.

REFERENCES :

1. S.S. Sastry : Introductory methods of Numerical analysis, Prentice- Hall of India, New Delhi (1998).
2. M.K. Jain, S.R.K. Iyengar and R.K. Jain : Numerical methods for scientific and Engineering computation, Wiley Eastern (1993)
3. B.K Kolman, R.C Busby and S. Ross, Discrete Mathematical Structure, PHI.
4. K.D Joshi, Foundations of Discrete Mathematics, Wiley Estern.

SEMESTER – IV

4.1 FUNCTIONAL ANALYSIS

Unit-1

Functional Analysis Norm on a linear space over F (either R or C), Banach space, Examples. Norm on Quotient space, Continuous Linear Transformation of normed linear space. The Banach space B(N,N') for Banach spaces , N, N'.

Unit-2

Dual space of normed linear space , Equivalence of norms, Dual space of C[a,b], Isometric isomorphism.

Unit-3

Hahn-Banach theorem and its applications, Seperable normed linear space

Unit-4

Canonical embedding of N into N**. Reflexive spaces, Open mapping theorem , Closed graph theorem, Principle of Uniform boundedness(Banach-Steinhauss Theorem), Projection on Banach spaces. Hilbert spaces, Definition and examples, Orthogonal complements, Orthonormal basis, Gram-Schimdt process of orthonormalisation, Besel's inequality, Riesz Fisher Theorem.

Unit-5

Adjoint of an operator, Self adjoint, normal, unitary and projection operators.

REFERENCES:

1. G.F.Simmons: Introduction to T opology and Modern Analysis, McGraw Hill Book company Inc (1962)
2. C.Goffman and G.Pedrick: First Course in Functional Analysis, Prentice Hall of India Pvt
3. Ltd N.Delhi(1974)
4. B.V.Limaye: Functional Analysis 2nd Edition, New Age International (P) Ltd
5. Publication 1997.
6. D.Somasundaram, Functional Analysis, S.Vishwanathan Printers and Publishers Pvt, Limited (1994)
7. Ponnuswamy, Foundations of Functional analysis, Narosa.
8. K.Chandrashekara Rao,Functional Analysis, Narosa

4.2 MATHEMATICAL METHODS

Unit -1

Integral Transforms:. Applications of Laplace transforms, Laplace transforms to solve ODEs and PDEs - typical examples. Integral Equations: General definition of Integral transforms, Kernels, etc. Definition, Volterra and Fredholm integral equations. Solution by separable kernel, Neumann's series,

Unit -2

Resolvant kernel and transform methods, Convergence for Fredholm and Volterra types. Reduction of IVPs, BVPs and eigen value problems to integral equations. Hilbert Schmidt theorem, Raleigh Ritz and Galerkin methods. Asymptotic Methods: Asymptotic expansion of functions, power series as asymptotic series, Asymptotic forms for large and small variables. Uniqueness properties and Operations.

Unit -3

Asymptotic expansions of integrals; Method of integration by parts (include examples where the method fails), Laplaces method and Watson's lemma, method of stationary phase and steepest descent.

Unit -4

Regular and singular perturbation methods: Parameter and co-ordinate perturbations. Regular perturbation solution of first and second order differential equations involving constant and variable coefficients. Include Duffings equation, Vanderpol oscillator, small Reynolds number flow.

Unit -5

Singular perturbation problems, Matched asymptotic expansions, simple examples. Linear equation with variable coefficients and nonlinear BVP's. Problems involving Boundary layers. Poincare – Lindstedt method for periodic solution. WKB method.

REFERENCE BOOKS

1. I.N. Sneddon – The use of Integral Transforms, Tata Mc Graw Hill, Publishing Company Ltd, New Delhi, 1974
2. R.P. Kanwal: Linear integral equations theory and techniques, Academic Press, New York, 1971
3. C.M. Bender and S.A. Orszag – Advanced mathematical methods for scientists and engineers, Mc Graw Hill, New York, 1978
4. H.T. Davis – Introduction to nonlinear differential and integral equations, Dover Publications, 1962.

4.3 PROBABILITY THEORY

Unit 1

Random Experiments, Sample spaces, Sets, Events, Algebras, Elements of combinatorial analysis, Classical definition and calculation of Probability, Independence of events

Unit 2

Random variables, Distribution functions, Moments, Probability and moment generating functions, Independence of random variables, Theoretical distributions: Binomial, Poisson and Normal distribution and their properties.

Unit 3

Correlation and Regression: Definition meaning scatter diagram method, Karl Pearson's method, Probable error, Standard error and Rank correlation. Regression: Definition: meaning two lines of regression, regression coefficients, standard error and relation between correlation and regression.

Unit 4

Introduction to various discrete and continuous random variables, Limiting distributions of some random variables Distributions of functions of random variables, Bi-variate distributions, Conditional and marginal distributions, Conditional expectation and variance, Co-variance and correlation co-efficient.

Unit 5

Elementary understanding of data: Frequency curves, Empirical measures of location, spread, empirical moments, Analysis of Bi-variate data, fitting of distributions.

REFERENCES:

1. Siva Athreya, V.S. Sunder, Measure and Probability, CRC Press
2. William Feller, Introduction to Probability and its applications, Vol - I, 3rd Edition
3. F.M.Dekking, C. Kraaikamp and others, A Modern introduction to Probability and Statistics, Springer Publication.
4. Athanasios Papoulies, Unnikrishna Pillai, Probability, random Variables and Stochastic Processes, Tata McGraw Hill.
5. K.B.Athreya, B.K.Lahiri, Measure Theory and Probability Theory, Hindustan BookAgency, TRIM Series.

4.4 ELECTIVE

I - RIEMANNIAN GEOMETRY

Unit1:

Local Geometry of Surfaces, First and Second fundamental form, Gaussian, Mean and normal curvatures, Geodesic Curvature Gauss Theorema Egregium.

Unit2:

Global Geometry of Surface, Geodesic coordinate patches, Gauss-Bonnet formula and Euler Characteristic. Index of a vector field. Surfaces of const curvature.

Unit3:

Concept of a tensor. Covariant differentiation. Symmetric Properties of curvature tensor.

Unit4:

Notion of affine connections, Parallel Transport Christofell Symbols of Ist and IInd kind.

Unit5:

Riemannian metric, affine connections associated with Riemannian metric, geodesics and normal coordinates.

REFERENCES:

1. W.Boothby, Differentiable Manifolds and Riemannian |Geometry,
2. Riemannian Geometry, M.Docarmo.
3. Laugwitz D. Differential and Riemannian Geometry, Acad Press, 1965.
4. Millman, RS, Parker G.D, Elements of Differential Geometry, PHI, 1977.
5. S. Helgason, Differential Geometry, Lie Groups and Symmetric Spaces, AMS Publishers.

4.4 ELECTIVE

II- GRAPH THEORY

Unit-1

Coverings, Vertex covering, Edge covering, Independence number, Matching and Matching polynomials, Factorization of graphs: Factorization-1 factorization, 2-factorization, and decomposition of Graphs.

Unit-2

Distance in Graphs: The centre of a graph, distant vertices. Colorings, Chromatic numbers and chromatic polynomials,

Unit-3

Spectra of Graphs: Adjacency matrix, incidence matrix, Characteristic polynomial, Eigen values, Energy of graphs: Energy of all standard class of graphs, Bonds of r energy of a graph.

Unit-4

Groups and Graphs, Automorphism group of Graph. Operations on Permutation graphs, The Group of composite graphs, Domination: Dominating sets, Domination numbers, Domatic number and its bonds, independent domination of a number of a Graph, Other domination parameters.

Unit-5

Topological indices of graphs: Degree based Topological indices: Randic index, Zagreb indices, reformulated Zagreb index and related bounds. Distance Based Topological Indices: Wiener index, Hyper- Wiener index and Harary index, related bounds.

REFERENCES:

1. G.Charatrand and Ping Zhang: Introduction to graph theory.
2. R.B.Bapat, Graphs and matrices.
3. I. Gutaman and Xi Li, Graph Energy.
4. I. Gutman and O.Polansky, Mathematical Concepts in organic Chemistry.
5. J.A.Bondy ans V.S.R.Murthy, Graph Theory with applications McMillan, London.
6. F. Buckley and F.Harary: Distance in Graphs, Addison Wesley, 1990.
7. Diestel: Graph Theory, Springer Verlag, Berlin
8. R.Gould: Graph Theory, Benjamin Cummins Publication Company Inc, Calif 1998.
9. F. Harary Graph theory, Addision Wesley, Reading Mass 1969.
10. O. Ore: theory of Graphs, Amer. Math, Soc. College Publications- 38 Providence 1962.
11. D. Cvetkovic, M.Doob and H.Sachs, Spectra in Graphs, Academic Press, 1980.

4.4 ELECTIVE

III - MATHEMATICAL MODELING

Unit 1

Some basic topics in Nonlinear Waves: Shock waves and hydraulic jumps. Description and various physical set ups where they occur: traffic flow, shallow water.

Unit 2

Fundamental concepts in continuous applied mathematics. Continuum limit. Conservation laws, quasi-equillibrium. Kinematic waves. _ Traffic flow (TF). Continuum hypothesis. Conservation and derivation of the mathematical model.

Unit 3

Integral and differential forms. Other examples of systems where conservation is used to derive the model equations (in nonlinear elasticity, fluids, etc.), Linearization of equations of TF and solution. Meaning and interpretation. Solution of the fully nonlinear TF problem.

Unit 4

Method of characteristics, graphical interpretation of the solution, wave breaking. Weak discontinuities, shock waves and rarefaction fans. Envelope of characteristics. Irreversibility in the model.

Unit 5

Quasilinear First Order PDE's, Shock structure, diffusivity. Burger's equation. The Cole-Hopf transformation. The heat equation: derivation, solution, and application to the Burger's equation. Inviscid limit and Laplace's method.

REFERENCES:

1. R. Haberman, Mathematical Models, Mechanical Vibrations, Population Dynamics and Traffic flow, SIAM.
2. C. C. Lin and L. Segal, Mathematics Applied to Deterministic Problems in the Natural Sciences, SIAM.
3. F. Y. M. Wan, Mathematical Models and their Analysis, Harper and Row.
4. J. D. Logan, An Introduction to Nonlinear Partial Differential Equations, J. Wiley.
5. R. D. Richtmyer and K. W. Morton Difference Methods for Initial-Value Problems, Inter science, Wiley, Krieger.
6. C. Fowler, Mathematical Models in the Applied Sciences, Cambridge U. Press.
7. J. J. Stoker, Nonlinear Vibrations in Mechanical and Electrical Systems , J. Wiley.
8. G. B. Whitham, Linear and Nonlinear Waves, J. Wiley.
9. R. Haberman, Applied Partial Differential Equations With Fourier Series and Boundary Value Problems, Prentice Hall.

4.4 ELECTIVE

IV - GALOIS THEORY

Unit 1

Field extensions, Characteristic of a Field, Finite fields, splitting field of a polynomial.

Unit 2

Algebraic extensions, Algebraic closure, algebraically closed field, Separable Extension, Simple extension, Primitive element theorem.

Unit 3

Inseparable extension, Purely inseparable extension, Perfect field, Imperfect field, Normal Extension Group of automorphisms of Field extensions.

Unit 4

Linear independence of characters, Artin's Theorem, Norm and Trace, Cyclic extension, Hilbert Theorem 90, Artic Schreier Theorem

Unit 5

Solvable extension, Solvability by Radicals, Insolvability of the Quintic, Theorem of Abel – Ruffini. Galois groups of quadratic, cubic and quartic polynomials over the rational field

REFERENCES:

1. J.J.Rotman, Galois Theory, Univeritext, Springer 1990.
2. D.J.H.Garling, A Course in Galois Theory CUP,1986
3. Ian Stewart, Galois Theory, Chapman and Hall, London, NewYork.
4. I.N.Herstein, Topics in Algebra Blaisidel, NY.
5. Sulrjeet Singh and Quazi Zameerudin, Modern Algebra. Vikas Publications.

4.5. ELECTIVE

I – ADVANCED NUMERICAL

ANALYSIS. Unit-1

Numerical solution of ordinary differential equations: Initial value problems - Picard's and Taylor series methods – Euler's Method- Higher order Taylor methods- Modified Euler's method- Runge Kutta methods of second and fourth order.

Unit-2

Multistep method- The Adams - Moulton method- stability- (Convergence and Truncation error for the above methods). Boundary- Value problems – Second order finite difference method, cubic spline method and shooting method.

Unit-3

Finite difference methods for Parabolic equations in one-dimension – methods of Schmidt, Laarsonen, Crank-Nicolson and Dufort. Frankel. Stability and convergence analysis for Schmidt and Crank-Nicolson methods and iterative methods.

Unit-4

A.D.I. method for two - dimensional parabolic equation. Finite difference methods for hyperbolic equations in one-dimension explicit and implicit finite difference schemes. Stability and convergence analysis for hyperbolic equations.

Unit-5

Numerical solution of Partial differential equations: Difference methods for Elliptic partial differential equations – Difference schemes for Laplace and Poisson's equations. Iterative methods of solution by Jacobi and Gauss-Seidel methods – solution techniques for rectangular and quadrilateral regions.

TEXT BOOKS

1. M.K. Jain: Numerical solution of differential equations, Wiley Eastern (1979), Second Edition.
2. C.F. Gerald and P.O. Wheatley : Applied Numerical Methods, Low- priced edition, Pearson Education Asia (2002), Sixth Edition.
3. D.V. Griffiths and I.M. Smith, Numerical Methods for Engineers, Blackwell Scientific Publications (1991).

REFERENCE BOOKS

1. S.C. Chapra, and P.C. Raymond : Numerical Methods for Engineers, Tata Mc Graw Hill, New Delhi (2000)
2. R.L. Burden, and J. Douglas Faires : Numerical Analysis, P.W.S. Kent Publishing Company, Boston (1989), Fourth edition.
3. S.S. Sastry : Introductory methods of Numerical analysis, Prentice- Hall of India, New Delhi (1998).
4. M.K. Jain, S.R.K. Iyengar and R.K. Jain : Numerical methods for scientific and Engineering computation, Wiley Eastern (1993)
5. G.D.Smith: Numerical Solutions of partial differential equations 2^{nd} edition London, Oxford University Press (1978)
6. Paruiz Moin: Fundamentals of Engineering Numerical analysis, Cambridge University Press (2006)

4.5 ELECTIVE

II –BANACH ALGEBRA

Unit 1

Preliminaries, Banach spaces, Weak topologies on Banach spaces, Banach valued functions and their derivatives, Holomorphic functions, Banach space values measures and Integration.

Unit 2

Definition of Banach Algebra, Homomorphisms, Spectrum, Basic properties of Spectra, Gelfand- Mazur Theorem, Spectral Mapping Theorem, group of invertible elements.

Unit 3

Ideals, Maximal Ideals and Homomorphisms, Semisimple Banach Algerbras

Unit 4

Gelfand Topology, Gelfand Transform, Involutions, Banach-C-*Algebras, Gelfand Naimark Theorem, Applications to Non-Commutative Banach Algebras, Positive functions.

Unit 5

Operators on Hilbert Spaces, Commutativity theorem, Resolution of the identity spectral theorem, A Charecterization of Banach C*-Algebras

REFERENCES

1. Rudin.W, Functional Analysis.
2. Bachman and Narice L, Functional Analysis , Academic Press.
3. B.V.Limaye, Functional Analysis, New Age International Limited
4. S.K.Berbenon, Lectures in Functional Analysis and Operator Theory, Narosa, 1979.

4.5 ELECTIVE

III - OPERATIONS

RESEARCH Unit-1

The linear programming problem, properties of a solution to the linear programming problem, generating extreme point solution, simplex computational procedure, development of minimum feasible solution, the artificial basis techniques, a first feasible solution using slack variables, two phase and Big-M method with artificial variables.

Unit-2

General Primal-Dual pair, formulating a dual problem, primal-dual pair in matrix form, Duality theorems, complementary slackness theorem, duality and simplex method, economic interpretation of duality, dual simplex method.

Unit-3

General transportation problem, transportation table, duality in transportation problem, loops in transportation tables, LP formulation, solution of transportation problem, test for optimality, degeneracy, transportation algorithm (MODI method), time- minimization transportation problem.

Unit-4

Mathematical formulation of assignment problem, assignment method, typical assignment problem, the traveling salesman problem. Game Theory: Two-person zero-sum games, maximin minimax principle, games without saddle points(Mixed strategies), graphical solution of $2\times n$ and $m\times 2$ games, dominance property, arithmetic method of $n\times n$ games, general solution of $m\times n$ rectangular games.

Unit-5

Integer Programming: Gomory's all I.P.P. method, constructions of Gomory's constraints, Fractional cut method-all integer and mixed integer, Branch-and-Bound method, applications of integer programming. Dynamic Programming: The recursive equation approach, characteristics of dynamic programming, dynamic programming algorithm, solution of-Discrete D.P.P., some applications, solution of L.P.P. by Dynamic Programming.

REFERENCES

1. Taha, Operations Research, Pearson Education; Eighth edition (2011)
2. Kambo N. S., Mathematical Programming,
3. G. Hadley, Linear Programming, Addison Wesley.
4. Gass, S. L., Linear Programming, Courier Dover Publications, 2003

4.5 ELECTIVE

IV- COMPUTATION COMPLEXITY

Unit1.

Turing machines; determinism and non-determinism --- time and space hierarchy theorems; speed-up and tape compression;

Unit2.

Blum axioms --- structure of complexity classes NP, P, NL, L, PSPACE;

Unit3.

Complete problems --- randomness and complexity classes RP, RL, BPP --- alternation, polynomial-time hierarchy --- circuit complexity --- parallel complexity, NC, RNC ---

Unit4.

Relativized computational complexity --- time-space trade-offs ---

Unit5.

Introduction to Interactive Proofs --- Arthur-Merlin Games, IP = PSPACE.

Reference Books:

1. C. H. Papadimitrou: Computational Complexity, Addison Wesley.
2. J. Radhkrishnan, S. Saluja: Interactive Proof Systems, TCS lecture notes, TIFR.

4.6 PROJECT

The candidate shall submit a dissertation carrying 80 marks and appear for viva-voce carrying 20 marks.